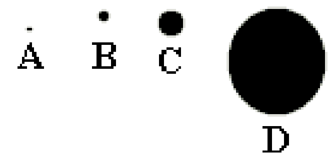
Basic Geometry Review

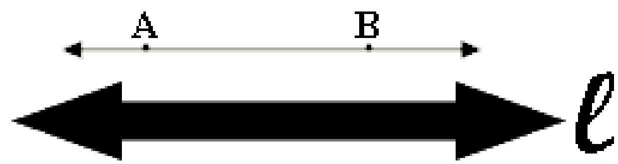
For Trigonometry Students

Undefined Geometric Terms

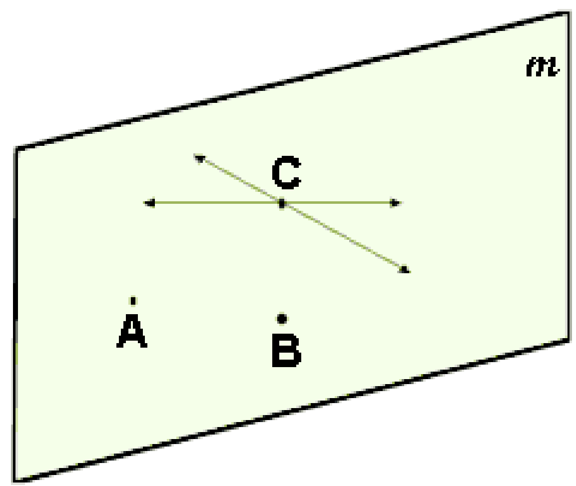
- Point *A*



- Line $\overleftrightarrow{AB}$



- Plane *ABC*



Half-lines (Rays)

- This is ray $\overrightarrow{AB}$

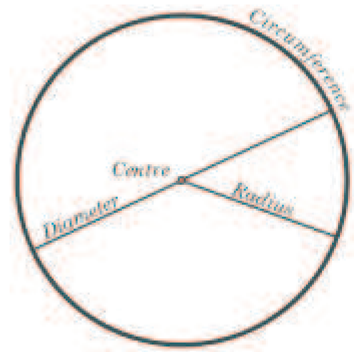
- Point A is the "vertex" or "endpoint" of $\overrightarrow{AB}$; always write the name of the endpoint first
- Definition: $\overrightarrow{AB}$ is the set of all points C on $\overleftrightarrow{AB}$ such that A is not strictly between B and C

Line Segments

- The green portion is line segment $\overline{AB}$

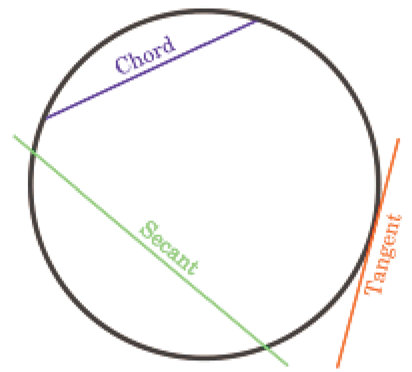
- Points *A* & *B* are "endpoints"; distance between them is *AB*
- Definition: $\overline{AB}$ is bounded by endpoints *A* and *B*; it contains every point on $\overleftrightarrow{AB}$ that is between endpoints *A* and *B*

Circles (1 of 5)



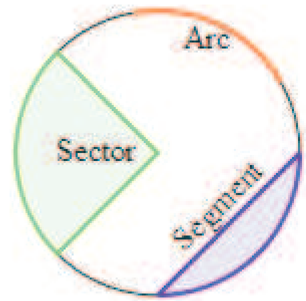
- Definition: A circle is the set of all points lying in a plane at a fixed distance *r* (the "radius") from a given point (the "center" of the circle)
- A "diameter" *d* is any line segment whose endpoints lie on the circle, and which passes through (contains) the center of the circle

Circles (2 of 5)



- A "secant line" is any line that touches the circle at exactly two points
- A "tangent line" is any line that touches the circle at exactly one point
- A "chord" is any line segment whose endpoints lie on the circle, but which does not pass through the exact center of the circle

Circles (3 of 5)



- The "circumference" is the full outer edge of the circle, or the length of it
- An "arc" is any continuous portion of the circumference
- A "sector" is the wedge-like shape bounded by two radii and the arc that lies between them
- A "segment" is the shape formed by a chord and the arc that extends between its endpoints

Circles (4 of 5)

- Formulas:
  - Diameter: $d = 2r$
  - Circumference: $C = 2\pi r = \pi d$
  - Area: $A = \pi r^2$
  - “Pi”: $\pi \approx 3.14159265358979\ldots$

Circles (5 of 5)

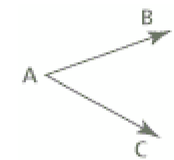
- Equations and unit circles
  - The equation of a circle whose center is located at the origin of a Cartesian coordinate system is

    $$x^2 + y^2 = r^2$$

  - A “unit circle” is a circle that has a radius of one unit ($r = 1$)
  - So the equation of a *unit* circle whose center is located at the origin of a Cartesian coordinate system is

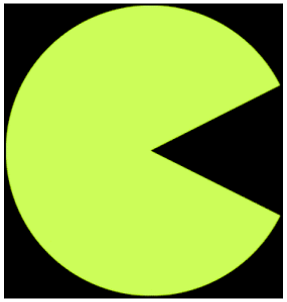
    $$x^2 + y^2 = 1$$

Angles



- An "angle" $\angle BAC$ (or $\angle CAB$ or $\angle A$, if the shorter name is clear) is the figure formed when two rays (the "sides" or "legs" of the angle) share a single endpoint A (the "vertex" of the angle); the vertex is always the middle letter
- Latin or Greek lowercase letters, such as a, b, θ, φ, α, or β, are also used to name angles in trigonometry and higher math

Angle Measure (1 of 3)



- Pac-Man's jaw forms an angle (the black wedge in the figure); the "measure" of the angle is a number that tells us about the size of the wedge (how far open Pac-Man's jaw has become)
- The angle's measure increases as Pac-Man opens up wider

Angle Measure (2 of 3)

- One unit often used to measure angles is the "degree" (symbol: °)
- Visit this web page* to learn about different kinds of angles:
  - Acute angles (measure $m < 90°$)
  - Right angles ($m = 90°$)
  - Obtuse angles ($90° < m < 180°$)
  - Straight angles ($m = 180°$)
  - Reflex angles ($180° < m < 360°$)

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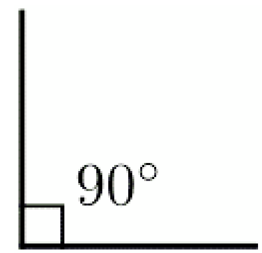
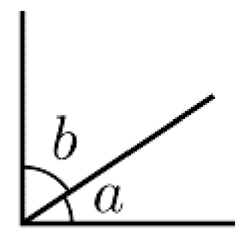
* http://www.mathopenref.com/angle.html

Angle Measure (3 of 3)

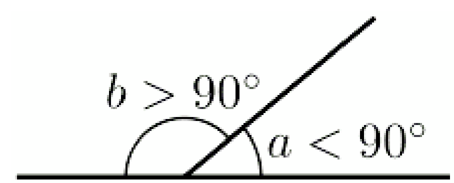
- If line segments, rays, or lines cross at a right angle ("perpendicular"), then a small square is often added to indicate this



- Two angles whose measures add up to 90° are "complementary"

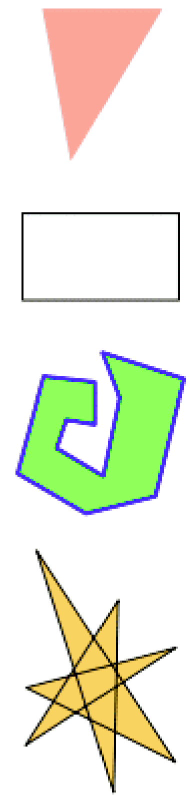


- Two angles whose measures add up to 180° are "supplementary"



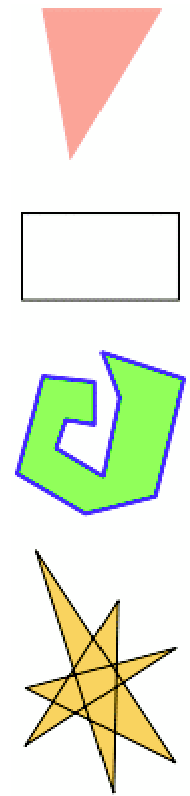
Polygons (1 of 5)

- The intuitive polygon:
  - Draw a random assortment of 3 or more points in a plane
  - Connect them so that each point is the endpoint of exactly two line segments, and no point lies on a given line segment unless it is one of that segment's two endpoints
  - The result is a "polygon" (some examples are shown in the figure at right)



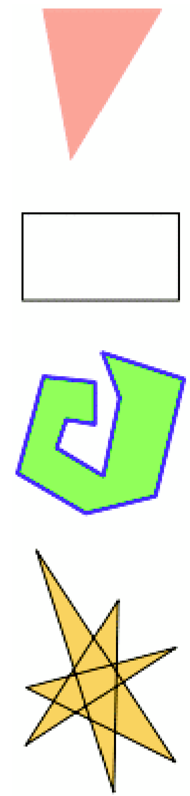
Polygons (2 of 5)

- The strictly defined polygon (you won't be tested on this): A "polygon" is a closed path composed of a finite sequence of straight line segments
- Other terms (you may be tested on these):
  - The line segments are called "sides" of the polygon
  - Each corner is called a "vertex" of the polygon



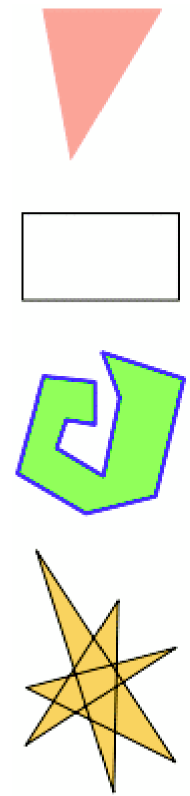
Polygons (3 of 5)

- "Polygons" are what an average person would call "shapes" … but there are some restrictions:
  - Polygons have no "curvy" parts; the definition (see the previous slide) requires each side to be straight
  - So, although circles, ellipses, parabolas, and other "curvy" things are called shapes also, they are *not* polygons



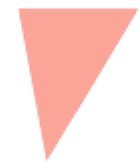
Polygons (4 of 5)

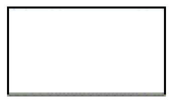
- Mathematicians classify polygons by the number of sides (or vertices) they have; the names used have mostly Greek roots:
  - 3 sides = “triangle” or “trigon”
  - 4 sides = “quadrilateral” or “tetragon”
  - 5 sides = “pentagon”
  - 6 sides = “hexagon”
  - 8 sides = “octagon”, *etc.*

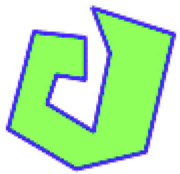


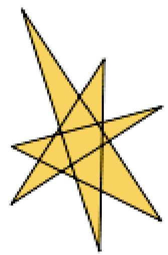
Polygons (5 of 5)

- Some polygons possess symmetry; terms used for certain types of symmetry include:
  - "Equiangular": All the vertex angles have equal measures
  - "Cyclic": All the vertices lie on a circle
  - "Equilateral": All the sides, or edges, have the same length
  - "Regular": The polygon is both cyclic and equilateral









Triangle Properties (1 of 2)

- The measures of the three vertex angles *always* add up to 180°
- An equilateral triangle is always equiangular (and vice-versa); if either of these is true, then both are true, and the measure of each vertex angle is exactly 60°
- An equilateral triangle is the only kind of triangle that is regular

Triangle Properties (2 of 2)

- If the lengths of at least two sides of a triangle are equal, then it is called an “isosceles triangle”
- If all three sides of a triangle have different lengths, then it is called a “scalene triangle”

Right Triangles

- If one vertex angle of a triangle is a right angle (has a measure of 90°), then the triangle is a “right triangle”, having these properties:
  - The two remaining vertex angles are automatically complementary
  - It may be either scalene or isosceles; if it is isosceles, then the two remaining vertex angles both have equal measures of exactly 45°
  - The Pythagorean theorem (Appendix A) relates the lengths of the 3 sides

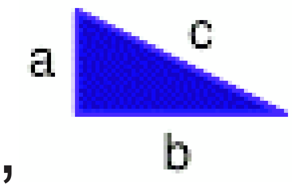
Quadrilateral Properties (1 of 2)

- The measures of the four vertex angles *always* add up to 360°
- An equilateral quadrilateral is called a “rhombus”; it is *not* necessarily equiangular or square
- An equiangular quadrilateral is called a “rectangle”; it is *not* necessarily equilateral
- All four vertices of a rectangle are right angles, and therefore have measures of 90°

Quadrilateral Properties (2 of 2)

- A “square” is a quadrilateral that is both equilateral and equiangular
- A square is the only kind of quadrilateral that is regular

Appendix A: Pythagorean Theorem



- If *c* is the length of the hypotenuse (longest side), and *a* and *b* are the lengths of the legs (shorter sides), then

$$a^2 + b^2 = c^2$$

  – The hypotenuse is always the side that does *not* touch the right angle

  – The figure depicts a scalene triangle; some right triangles might also be isosceles, but they can never be equilateral

Appendix B: The Greek Alphabet

- Α α alpha (άλφα)
- Β β beta (βήτα)
- Γ γ gamma (γάμμα)
- Δ δ delta (δέλτα)
- Ε ε epsilon (έψιλον)
- Ζ ζ zeta (ζήτα)
- Η η eta (ήτα)
- Θ θ,ϑ theta (θήτα)
- Ι ι iota (ιώτα)
- Κ κ kappa (κάππα)
- Λ λ lambda (λάμδα)
- Μ μ mu (μι)
- Ν ν nu (νι)
- Ξ ξ xi (ξι)
- Ο ο omicron (όμικρον)
- Π π pi (πι)
- Ρ ρ rho (ρω)
- Σ σ,ς sigma (σίγμα)
- Τ τ tau (ταυ)
- Υ υ upsilon (ύψιλον)
- Φ φ,ϕ phi (φι)
- Χ χ chi (χι)
- Ψ ψ psi (ψι)
- Ω ω,ϖ omega (ωμέγα)