



ESSENTIAL TRIGONOMETRIC IDENTITIES FOR PHYSICS & CALCULUS

For each real x , θ , α , and β that are elements of the domain of the specified functions, the following identities hold:

1. Definitions of derived trigonometric functions:

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}; \quad \cot \theta \equiv \frac{\cos \theta}{\sin \theta} \equiv \frac{1}{\tan \theta}; \quad \sec \theta \equiv \frac{1}{\cos \theta}; \quad \csc \theta \equiv \frac{1}{\sin \theta}$$

2. Even/odd properties of trigonometric functions:

$$\begin{aligned} \cos(-\theta) &\equiv \cos(\theta); & \sin(-\theta) &\equiv -\sin(\theta); & \tan(-\theta) &\equiv -\tan(\theta) \\ \cot(-\theta) &\equiv -\cot(\theta); & \sec(-\theta) &\equiv \sec(\theta); & \csc(-\theta) &\equiv -\csc(\theta) \end{aligned}$$

3. Identities obtained from $\sin^2 \theta + \cos^2 \theta = 1$:

$$\tan^2 \theta + 1 \equiv \sec^2 \theta \qquad \cot^2 \theta + 1 \equiv \csc^2 \theta$$

4. Sums or differences of angles:

$$\cos(\alpha \pm \beta) \equiv \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin(\alpha \pm \beta) \equiv \sin \alpha \cos \beta \pm \sin \beta \cos \alpha$$

$$\tan(\alpha \pm \beta) \equiv \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \qquad \cot(\alpha \pm \beta) \equiv \frac{\cot \alpha \cot \beta \mp 1}{\cot \beta \pm \cot \alpha}$$

$$\sec(\alpha \pm \beta) \equiv \frac{\sec \alpha \sec \beta}{1 \mp \tan \alpha \tan \beta} \qquad \csc(\alpha \pm \beta) \equiv \frac{\csc \alpha \csc \beta}{\cot \beta \pm \cot \alpha}$$

5. Double angle formulas:

$$\cos(2\theta) \equiv \cos^2 \theta - \sin^2 \theta \equiv 2\cos^2 \theta - 1 \equiv 1 - 2\sin^2 \theta$$

$$\sin(2\theta) \equiv 2\sin \theta \cos \theta$$

$$\tan(2\theta) \equiv \frac{2 \tan \theta}{1 - \tan^2 \theta} \qquad \cot(2\theta) \equiv \frac{\cot^2 \theta - 1}{2 \cot \theta}$$

$$\sec(2\theta) \equiv \frac{\sec^2 \theta}{1 - \tan^2 \theta} \qquad \csc(2\theta) \equiv \frac{\csc^2 \theta}{2 \cot \theta}$$

6. Sum-to-product/difference-to-product formulas:

$$\cos \alpha + \cos \beta \equiv 2 \cos \left[\frac{1}{2}(\alpha + \beta) \right] \cos \left[\frac{1}{2}(\alpha - \beta) \right]$$

$$\cos \alpha - \cos \beta \equiv 2 \sin \left[\frac{1}{2}(\beta - \alpha) \right] \sin \left[\frac{1}{2}(\beta + \alpha) \right]$$

$$\sin \alpha \pm \sin \beta \equiv 2 \sin \left[\frac{1}{2}(\alpha \pm \beta) \right] \cos \left[\frac{1}{2}(\alpha \mp \beta) \right]$$

$$\tan \alpha \pm \tan \beta \equiv \tan(\alpha \pm \beta)(1 \mp \tan \alpha \tan \beta)$$

7. Product-to-sum formulas:

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

8. Complementary angle and 90° rotation formulas:

$$\cos\left(\frac{\pi}{2} - \theta\right) \equiv \sin \theta \qquad \cos\left(\theta \pm \frac{\pi}{2}\right) \equiv \mp \sin \theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) \equiv \cos \theta \qquad \sin\left(\theta \pm \frac{\pi}{2}\right) \equiv \pm \cos \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) \equiv \cot \theta \qquad \tan\left(\theta \pm \frac{\pi}{2}\right) \equiv -\cot \theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) \equiv \tan \theta \qquad \cot\left(\theta \pm \frac{\pi}{2}\right) \equiv -\tan \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) \equiv \csc \theta \qquad \sec\left(\theta \pm \frac{\pi}{2}\right) \equiv \mp \csc \theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) \equiv \sec \theta \qquad \csc\left(\theta \pm \frac{\pi}{2}\right) \equiv \pm \sec \theta$$

9. Supplementary angle and 180° rotation formulas:

$$\cos(\pi - \theta) \equiv -\cos \theta \qquad \cos(\theta \pm \pi) \equiv -\cos \theta$$

$$\sin(\pi - \theta) \equiv \sin \theta \qquad \sin(\theta \pm \pi) \equiv -\sin \theta$$

$$\tan(\pi - \theta) \equiv -\tan \theta \qquad \tan(\theta \pm \pi) \equiv \tan \theta$$

$$\cot(\pi - \theta) \equiv -\cot \theta \qquad \cot(\theta \pm \pi) \equiv \cot \theta$$

$$\sec(\pi - \theta) \equiv -\sec \theta \qquad \sec(\theta \pm \pi) \equiv -\sec \theta$$

$$\csc(\pi - \theta) \equiv \csc \theta \qquad \csc(\theta \pm \pi) \equiv -\csc \theta$$

10. Squares of trigonometric functions:

$$\cos^2 \theta \equiv \frac{1}{2} + \frac{1}{2} \cos 2\theta \qquad \sin^2 \theta \equiv \frac{1}{2} - \frac{1}{2} \cos 2\theta$$

$$\tan^2 \theta \equiv \frac{1 - \cos 2\theta}{1 + \cos 2\theta} \qquad \cot^2 \theta \equiv \frac{1 + \cos 2\theta}{1 - \cos 2\theta}$$

$$\sec^2 \theta \equiv \frac{2}{1 + \cos 2\theta} \qquad \csc^2 \theta \equiv \frac{2}{1 - \cos 2\theta}$$

11. Half angle formulas:

$$\cos \frac{\theta}{2} \equiv \pm \sqrt{\frac{1}{2}(1 + \cos \theta)} \qquad \sin \frac{\theta}{2} \equiv \pm \sqrt{\frac{1}{2}(1 - \cos \theta)}$$

$$\tan \frac{\theta}{2} \equiv \frac{\sin \theta}{1 + \cos \theta} \equiv \frac{1 - \cos \theta}{\sin \theta} \equiv \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

12. Compositions of trigonometric and inverse trigonometric functions (for $-1 \leq x \leq 1$ and $0 \leq \theta \leq \pi/2$):

$$\cos(\sin^{-1} x) \equiv \sqrt{1 - x^2} \qquad \cos(\tan^{-1} x) \equiv \frac{1}{\sqrt{1 + x^2}}$$

$$\cos^{-1}(\sin \theta) \equiv \frac{\pi}{2} - \theta \qquad \sin(\cos^{-1} x) \equiv \sqrt{1 - x^2}$$

$$\sin(\tan^{-1} x) \equiv \frac{x}{\sqrt{1 + x^2}} \qquad \sin^{-1}(\cos \theta) \equiv \frac{\pi}{2} - \theta$$

$$\tan(\cos^{-1} x) \equiv \frac{\sqrt{1 - x^2}}{x} \qquad \tan(\sin^{-1} x) \equiv \frac{x}{\sqrt{1 - x^2}}$$