Solution to Section 2.6, problem 26

$$\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)=\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)\left[\frac{x-\sqrt{x^2+2x}}{x-\sqrt{x^2+2x}}\right]=\lim_{x\to-\infty}\frac{x^2-\left(x^2+2x\right)}{x-\sqrt{x^2+2x}}$$

$$=\lim_{x\to-\infty}\frac{-2x}{x-\sqrt{x^2(1+\frac{2}{x})}}=\lim_{x\to-\infty}\frac{-2x}{x-\sqrt{x^2}\sqrt{(1+\frac{2}{x})}}$$

Now we must think carefully. It is natural to write that $\sqrt{x^2}=x$, but actually $\sqrt{x^2}=|x|$ when we take into account the possibility that x might be negative. Since we are taking the limit as x approaches *negative* infinity, we conclude that x is indeed negative in this case, and so $|x|=-x$ (that is, in taking the absolute value of an inherently negative x, we obtain a result whose sign is the opposite of the number inside the absolute value symbol). So in the above equation, we replace $\sqrt{x^2}$ with $-x$, not x. Continuing with the above derivation, we obtain

$$\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)=\lim_{x\to-\infty}\frac{-2x}{x-(-x)\sqrt{(1+\frac{2}{x})}}=\lim_{x\to-\infty}\frac{-2x}{x+x\sqrt{(1+\frac{2}{x})}}$$

$$=\lim_{x\to-\infty}\frac{-2x}{x\left(1+\sqrt{(1+\frac{2}{x})}\right)}=\lim_{x\to-\infty}\frac{-2}{\left(1+\sqrt{(1+\frac{2}{x})}\right)}$$

At this point, it is possible to solve the limit by direct substitution, as it is well known that $\lim_{x\to-\infty}\frac{2}{x}=0$. We obtain

$$\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)=\frac{-2}{\left(1+\sqrt{(1+0)}\right)}=\frac{-2}{1+\sqrt{1}}=\frac{-2}{1+1}=\frac{-2}{2}=\underline{\underline{-1}}$$

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