Basic Geometry Review
For Trigonometry Students
Undefined Geometric Terms

Point $A$

Line $\overline{AB}$

Plane $ABC$
Half-lines (Rays)

- This is a ray named $\overrightarrow{AB}$
- Point $A$ is the “vertex” or “endpoint” of the ray; write the name of the endpoint first
- Definition: $\overrightarrow{AB}$ is the set of all points $C$ on $\overrightarrow{AB}$ such that $A$ is not strictly between $B$ and $C$
Line Segments

- The **green** portion is line segment $\overline{AB}$
- Points $A$ and $B$ are “endpoints”; the distance between them is written $AB$ (without the line segment over it)
- Definition: $\overline{AB}$ is bounded by endpoints $A$ and $B$; it contains every point on $\overline{AB}$ that is between endpoints $A$ and $B$
Circles (1 of 5)

- Definition: A circle is the set of all points lying in a plane at a fixed distance \( r \) (the “radius”) from a given point (the “center” of the circle)
- A “diameter” \( d \) is any line segment whose endpoints lie on the circle, and which passes through (contains) the center of the circle
Circles (2 of 5)

- A “secant line” is any line that touches the circle at exactly two points.
- A “tangent line” is any line that touches the circle at exactly one point.
- A “chord” is any line segment whose endpoints lie on the circle, but which does not pass through the exact center of the circle.
Circles (3 of 5)

- The “circumference” is the full outer edge of the circle, or the length of it.
- An “arc” is any continuous portion of the circumference.
- A “sector” is the wedge-like shape bounded by two radii and the arc that lies between them.
- A “segment” is the shape formed by a chord and the arc that extends between its endpoints.
Circles (4 of 5)

**Formulas:**
- **Diameter:** \( d = 2r \)
- **Circumference:** \( C = 2\pi r = \pi d \)
- **Area:** \( A = \pi r^2 \)
- **“Pi”:**
  - \( \pi \approx 3.141592653589793238… \)
  - \( \pi = \frac{C}{d} \)
Circles (5 of 5)

- Equations and unit circles
  - The equation of a circle whose center is located at the origin of a Cartesian coordinate system is
    \[ x^2 + y^2 = r^2 \]
  - A “unit circle” is a circle that has a radius of one unit \((r = 1)\)
  - So the equation of a unit circle whose center is located at the origin of a Cartesian coordinate system is
    \[ x^2 + y^2 = 1 \]
Angles

- An “angle” $\angle BAC$ (or $\angle CAB$ or $\angle A$, if the shorter name is clear) is the figure formed when two rays (the “sides” or “legs” of the angle) share a single endpoint $A$ (the “vertex” of the angle); the vertex is always the middle letter.

- Latin or Greek lowercase letters, such as $a$, $b$, $\theta$, $\varphi$, $\alpha$, or $\beta$, are also used to name angles in trigonometry and higher math.
Angle Measure (1 of 3)

- Pac-Man’s jaw forms an angle (the black wedge in the figure); the “measure” of the angle is a number that tells us about the size of the wedge (how far open Pac-Man’s jaw has become)
- The angle’s measure increases as Pac-Man opens up wider
Angle Measure (2 of 3)

- One unit often used to measure angles is the “degree” (symbol: °)
- Visit this web page* to learn about different kinds of angles:
  - Acute angles (measure $m < 90°$)
  - Right angles ($m = 90°$)
  - Obtuse angles ($90° < m < 180°$)
  - Straight angles ($m = 180°$)
  - Reflex angles ($180° < m < 360°$)

* http://www.mathopenref.com/angle.html
Angle Measure (3 of 3)

- If line segments, rays, or lines cross at a right angle ("perpendicular"), then a small square is often added to indicate this.

- Two angles whose measures add up to 90° are "complementary".

- Two angles whose measures add up to 180° are "supplementary".
The intuitive polygon:

- Draw a random assortment of 3 or more points in a plane
- Connect them so that each point is the endpoint of exactly two line segments, and no point lies on a given line segment unless it is one of that segment’s two endpoints
- The result is a “polygon” (some examples are shown at right)
The strictly defined polygon (you won’t be tested on this): A “polygon” is a closed path composed of a finite sequence of straight line segments.

Other terms (you may be tested on these):
- The line segments are called “sides” of the polygon.
- Each corner is called a “vertex” of the polygon.
“Polygons” are what most people would call “shapes” … but there are some restrictions:

- Polygons have no “curvy” parts; the definition (see the previous slide) requires each side to be straight.
- So, although circles, ellipses, parabolas, and other “curvy” things are called shapes also, they are not polygons.
Polygons (4 of 5)

- Mathematicians classify polygons by the number of sides (or vertices) they have; the names used have mostly Greek roots:
  - 3 sides = “triangle” or “trigon”
  - 4 sides = “quadrilateral” or “tetragon”
  - 5 sides = “pentagon”
  - 6 sides = “hexagon”
  - 8 sides = “octagon”, etc.
Some polygons possess symmetry; terms used for certain types of symmetry include:

- “Equiangular”: All the vertex angles have equal measures
- “Cyclic”: All the vertices lie on a circle
- “Equilateral”: All the sides, or edges, have the same length
- “Regular”: The polygon is both cyclic and equilateral
Triangle Properties (1 of 2)

- A “triangle” is a polygon that has 3 sides
- The measures of the three vertex angles always add up to 180°
- An equilateral triangle is always equiangular (and vice-versa); if either of these is true, then both are true, and the measure of each vertex angle is exactly 60°
- An equilateral triangle is the only kind of triangle that is regular
Triangle Properties (2 of 2)

- If the lengths of at least two sides of a triangle are equal, then it is called an “isosceles triangle”

- If all three sides of a triangle have different lengths, then it is called a “scalene triangle”
Right Triangles

- If one vertex angle of a triangle is a right angle (has a measure of 90°), then the triangle is a “right triangle”, having these properties:
  - The two remaining vertex angles are automatically complementary
  - It may be either scalene or isosceles; if it is isosceles, then the two remaining vertex angles both have equal measures of exactly 45°
  - The Pythagorean theorem (Appendix A) relates the lengths of the 3 sides
Quadrilateral Properties (1 of 2)

- The measures of the four vertex angles always add up to 360°
- An equilateral quadrilateral is called a “rhombus”; it is not necessarily equiangular or square
- An equiangular quadrilateral is called a “rectangle”; it is not necessarily equilateral
- All four vertices of a rectangle are right angles, and therefore have measures of 90°
Quadrilateral Properties (2 of 2)

- A “square” is a quadrilateral that is both equilateral and equiangular.

- A square is the only kind of quadrilateral that is regular.
Appendix A: Pythagorean Theorem

- If $c$ is the length of the hypotenuse (longest side), and $a$ and $b$ are the lengths of the legs (shorter sides), then
  
  $$a^2 + b^2 = c^2$$

  - The hypotenuse is always the side that does not touch the right angle
  - The figure depicts a scalene triangle; some right triangles might also be isosceles, but they can never be equilateral
Appendix B: Linear Measurements

- **English:**
  - 1 inch = 2.54 cm
  - 1 foot = 12 inches
  - 3 feet = 1 yard
  - 5280 feet = 1 mile

- **SI (metric):**
  - 1 m = 100 cm
  - 1 m = 1000 mm
  - 1 km = 1000 m
## Appendix C: The Greek Alphabet

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